Chapter 1

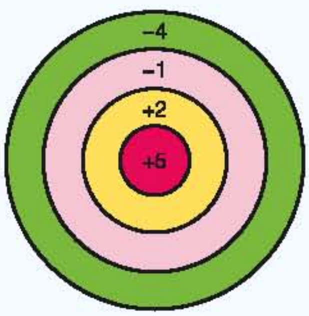
Directed Numbers

Learning Objectives

After completing this chapter, you will be able to

- understand and apply the concept of negative numbers.
- know the concept of the order of numbers on a number line.
- handle the operations of directed numbers.

The following figure shows a dartboard with 4 regions. The scores of the respective regions are +5, +2, −1 and −4 as shown. How can Patrick obtain a total score of 0 by throwing 3 darts onto the dartboard? (There may be more than one answer.)



Preview

[Basic techniques required for this chapter.]

Basic Technique

1. Compare the values of numbers

e.g. **(a)** Arrange $6\frac{1}{2}$, 0, 6, 3 in descending order.

i.e. $6\frac{1}{2} > 6 > 3 > 0$.

◀ '>' means 'larger than'.

(b) Arrange 1.5, 8, 7.5, 7 in ascending order.

i.e. $1.5 < 7 < 7.5 < 8$.

◀ '<' means 'smaller than'.

2. Order of the four basic operations of arithmetic

(a) Order of basic operations

Do multiplications and divisions before additions and subtractions from the left to the right.

e.g.
$$\begin{aligned} 10-7+2+24\div4\times3 &= 10-7+2+6\times3 \\ &= 10-7+2+18 \\ &= \underline{\underline{23}} \end{aligned}$$

(b) Evaluate the expressions with brackets

If there are many layers of brackets in an expression, we should handle the operations within the inner brackets first. Then evaluate the expression, layer by layer, from the inner brackets to the outer ones following the order of basic operations.

e.g.
$$\begin{aligned} 2\times[18-(6+3)\div3] &= 2\times(18-9\div3) \\ &= 2\times(18-3) \\ &= 2\times15 \\ &= \underline{\underline{30}} \end{aligned}$$

1.1 Positive and Negative Numbers

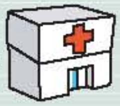
Class Activity 1.1

Aim: To learn numbers with opposite meanings

John's home is located at Lok See Road. There are different shops and facilities nearby. According to the figure below, answer the following questions.

1. If the fast food restaurant is located at 300 m to the left of John's home, write down the locations of the following places with respect to John's home.

Show Ans.

		Location
(a)	Fast food restaurant 	300 m to the left
(b)	Clinic 	____________
(c)	Bus stop 	____________
(d)	Library 	____________
(e)	Bank 	____________

2. Mary called John and told him that she was 300 m from his home at Lok See Road. Can you tell exactly her location?

Show Ans.

____

3. Are '200 m to the left' and '200 m to the right' at the same location? Show Ans.

☐ Yes ☐ No

Now I see ...

To express numbers with opposite meanings, we need to add descriptions to the *numerical values*.

numerical value 數值

From the above Class Activity, the numbers with descriptions in words like '200 m to the left' and '200 m to the right' are two numbers with opposite meanings. In Mathematics, we can express them as –200 m and +200 m respectively.

Numbers such as +200, +300 which carry a positive sign '+' are called **positive numbers**; numbers such as –200, –300 which carry a negative sign '–' are called **negative numbers**. Positive and negative numbers are opposite in meaning.

Note: 0 is neither a positive nor a negative number.

Additional Example 1.1

Example 1.1 Expressing quantities in positive or negative numbers or by words

(a) If '+' represents east and '–' represents west, express the following locations in positive or negative numbers.
  (i) 300 m to the east
  (ii) 400 m to the west

(b) If '+' represents a deposit and '–' represents a withdrawal, express the following *transactions* in positive or negative numbers.
  (i) A deposit of \$3 500
  (ii) A withdrawal of \$4 800

(c) If '+' represents arriving the country and '–' represents departing the country, describe the following changes in population by words.
  (i) –34 000 people
  (ii) +42 000 people

Solution

(a) (i) '300 m to the east' is expressed as +300 m.
  (ii) '400 m to the west' is expressed as –400 m.

(b) (i) 'A deposit of \$3 500' is expressed as +\$3 500.
  (ii) 'A withdrawal of \$4 800' is expressed as –\$4 800.

(c) (i) '–34 000 people' represents there are 34 000 people departing the country.
  (ii) '+42 000 people' represents there are 42 000 people arriving the country.

Classwork 1.1

Solution

Fill in the blanks.

(a) If +10 marks represents obtaining 10 marks, then ________ represents deducting 25 marks.

(b) If –\$30 represents paying \$30 by Octopus card , then ________ represents increasing the stored value of Octopus card by \$200.

(c) If +1 m represents 1 m above the sea level, then –40 m represents ________.

(d) If –2 people represents there are 2 students leaving school, then +3 people represents there are ________.

positive number 正數　negative number 負數　transaction 交易

Skills Upgrading Corner 1.1

Solution

The following are the passbook records of Mr. Chan's savings account.

Date	Withdrawal	Deposit / Interest	Balance
02 APR 07		*2,500.00	*42,800.00
15 APR 07	*1,900.00		*40,900.00
16 APR 07		*5,800.00	*46,700.00
30 APR 07	*3,250.00		*43,450.00
01 MAY 07		*35,000.00	*78,450.00

The table below shows the changes in the balance of the savings account.
Complete the table.

Date	Withdrawal / Deposit	Balance
02 APR 07		\$42 800
15 APR 07		\$40 900
16 APR 07		\$46 700
30 APR 07		\$43 450
01 MAY 07	+\$35 000	\$78 450

Exercise 1 A

Level 1

1. Fill in the blanks with positive or negative numbers.

(a) The room temperature is 21°C, and thus it is ______________.

(b) The weight of a dog increases by 2 kg. The change in its weight is ______________.

(c) The temperature is 15°C below zero, and thus it is ______________.

(d) The weight of Renee decreases by 1 kg. The change in her weight is ______________.

(e) A *submarine* moves down 100 m from the sea level. The change in its location is ______________.

submarine 潛艇

2. The sales records of Star Movie Club from January to July are as follows. Rewrite the table by using positive and negative numbers.

Month	January	February	March	April	May	June	July
Sales record	*Loss* \$20 000	Loss \$10 000	Loss \$15 000	*Profit* \$30 000	Profit \$20 000	Profit \$50 000	Loss \$10 000

3. If '+\$1' represents an increase of \$1 in the price of each bottle of cooking oil, express the following price changes in positive or negative numbers.

(a) An increase of \$5
(b) A decrease of \$7
(c) An increase of \$10
(d) A decrease of \$8

4. If '–1 m' represents walking 1 m to the south, express the following in positive or negative numbers.

(a) Walking 3 km to the south
(b) Walking 5 km to the north
(c) Walking 10 km to the south
(d) Walking 8 km to the north

5. Express the following water temperatures in positive or negative numbers.
(a) 45°C above the freezing point
(b) 43°C below the freezing point
(c) 73°C below the freezing point
(d) 20°C above the freezing point

The freezing point of water is 0°C.

6. May always borrows books from and returns them to the library. If '+1 book' represents that a book is borrowed by May, what is the meaning of '–23 books'?

7. Some watches are running a few minutes ahead or behind. If '+5 minutes' represents that a watch is running 5 minutes ahead, what is the meaning of '–20 minutes'?

Level 2

8. If '–1 hour' represents 1 hour before noon, express the following times in positive or negative numbers.
(a) 9 a.m. **(b)** 2 p.m.
(c) 3 a.m. **(d)** 11 p.m.

9. It is given that the school term will begin on 2nd September this year. If '–1 day' represents a day before the school term begins, i.e. 1st September, express the following dates in positive or negative numbers.
(a) 4th September **(b)** 30th August
(c) 11th September **(d)** 22nd August

loss 虧蝕 profit 盈利

10. The average weight of S1A students is 47 kg. The differences between the weights of 5 students of S1A and the average weight are as follows (let '+1 kg' be 1 kg more than the average weight):

Ken	Wing	Patrick	Wendy	Adam
+7 kg	0 kg	−9 kg	−2 kg	+3 kg

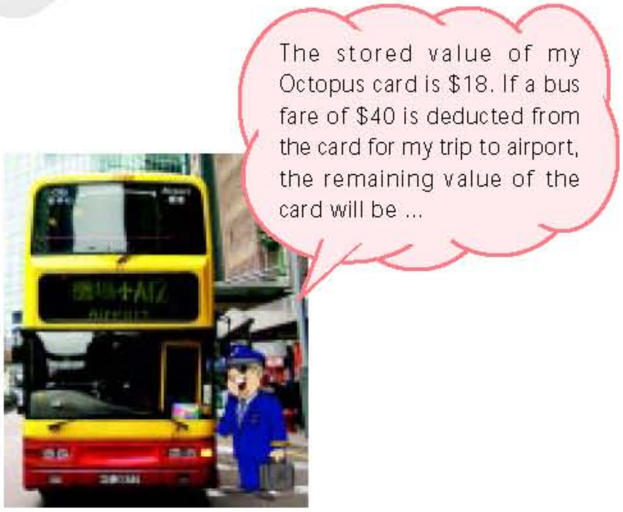
(a) What is the weight of Wendy?

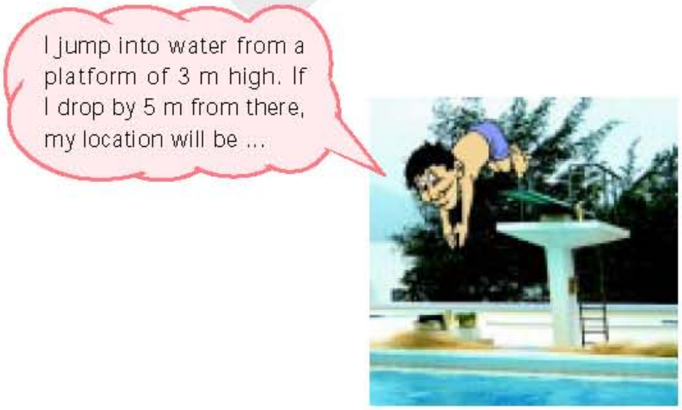
(b) Who is the heaviest? Who is the lightest? Explain briefly.

(c) It is given that Ron weighs 42 kg. Compare his weight with the average weight, and express the difference in a positive or negative number.

1.2 Directed Numbers and Number Lines

During primary education, the subtractions we dealt with involved subtracting a smaller number from a larger number. In fact, we often come across completely opposite situations in real life.





If negative numbers are also used to represent quantity, the above problems can be easily solved.

A Number lines

Positive and negative numbers can be shown by points on a straight line. This straight line is called a **number line**. On a number line, numbers are arranged in ascending order from the left to the right. Positive numbers are located on the right-hand side of zero and negative numbers are on the left-hand side of zero. (See Figure 1.1)

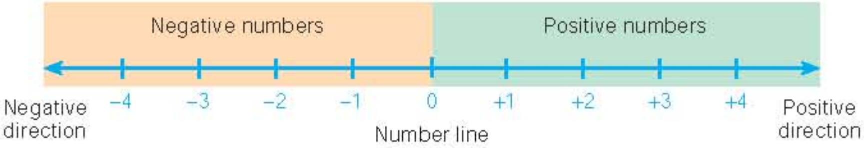


Figure 1.1

(a) Distances between any two successive integers are equal.

(b) The arrows at two ends are the signs for the unlimited extension of a number line to the two directions.

e.g. +6 means moving 6 units to the right from 0.

−6 means moving 6 units to the left from 0.

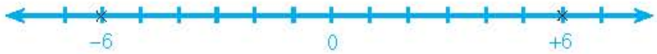


Figure 1.2

A **directed number** is a number with a positive or negative sign before it. It shows both the magnitude and the direction of a number.

Positive and negative numbers are on the opposite directions of zero. For positive numbers, their positive signs '+' are usually omitted. e.g. '+4' can be written as '4'.

Notes:

(a) On a number line, −6 is regarded as the opposite of +6. Thus, −6 is called the **opposite number** of +6 and it is denoted by $-(+6)=-6$. Similarly, +6 is regarded as the opposite of −6. Thus, +6 is called the opposite number of −6 and it is denoted by $-(-6)=+6$.

(b) A number line can also be placed vertically where all positive numbers are placed above 0 and all negative numbers are below 0. In addition, all numbers are arranged in ascending order from the bottom to the top.

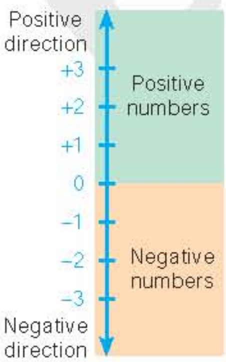


Figure 1.3

The markings on the thermometer form a number line.

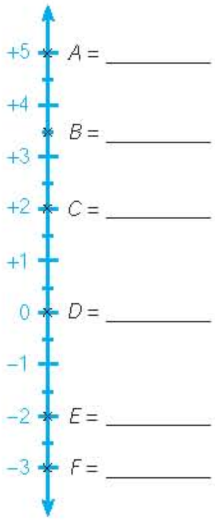
(c) In this book, horizontal number lines are used unless otherwise stated.

number line 數綫　　directed number 有向數　　opposite number 相反數

Extension 1.1

Solution

1. According to the number line below, find the values of points A to F.



Solution

2. Mark the following points A to F on the number line.

$A = -3$, $B = 0$, $C = 0.5$, $D = -0.5$, $E = 5$, $F = -5$

Solution

3. Complete the following.

(a) $-(+2) =$ __________ **(b)** $-(+5) =$ __________

(c) $-(-3) =$ __________ **(d)** $-(-7) =$ __________

B Order

On a number line, a larger number locates on the right of a smaller number.

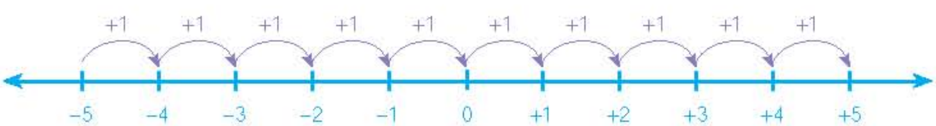


Figure 1.4

e.g. On a number line, a larger number +2 is on the right-hand side of a smaller number −3.

Conversely, on a number line, a smaller number locates on the left of a larger number.

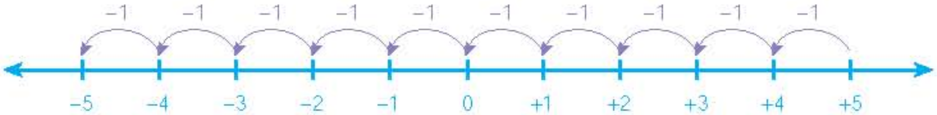


Figure 1.5

e.g. On a number line, a smaller number –3 is on the left-hand side of a larger number –1.

> On a number line, a number is greater than any numbers lying on its left. However, it is smaller than any numbers lying on its right.

Obviously, all positive numbers are greater than zero while all negative numbers are smaller than zero.

When comparing two numbers, we may find out their locations on a number line first. The number on the right-hand side must be greater than the one on the left-hand side.

Example 1.2 Comparing numbers

Additional Example 1.2

Compare each of the following groups of numbers, and fill in the blanks with '>' or '<'.

(a) $+7 \ \square \ +10$

(b) $+7 \ \square \ -10$

(c) $-5 \ \square \ +1$

(d) $-20 \ \square \ -60$

(e) $0 \ \square \ -100$

(f) $-15 \ \square \ 0$

Solution

(a)

$\therefore$ $\underline{\underline{+7 < +10}}$

'∴' means 'therefore'.

Classwork 1.2

Solution

Compare each group of numbers below, and fill in the blanks with '>' or '<'.

(a) $+1 \ \square \ +6$

(b) $+1 \ \square \ -1$

(c) $-9 \ \square \ +6$

(d) $-4 \ \square \ -8$

(e) $0 \ \square \ +18$

(f) $0 \ \square \ -5$

(b) 

∴ $\underline{\underline{+7 > -10}}$

(c) 

∴ $\underline{\underline{-5 < +1}}$

(d) 

∴ $\underline{\underline{-20 > -60}}$

(e) 

∴ $\underline{\underline{0 > -100}}$

(f) 

∴ $\underline{\underline{-15 < 0}}$

Additional Example 1.3

Example 1.3 Arranging numbers in order

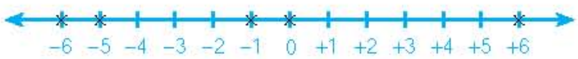
(a) Arrange the following numbers in descending order.
+5, –4, –3, +3, –1

(b) Arrange the following numbers in ascending order.
+6, –1, –6, 0, –5

Solution

(a) 

∴ $\underline{\underline{+5 > +3 > -1 > -3 > -4}}$

(b) 

∴ $\underline{\underline{-6 < -5 < -1 < 0 < +6}}$

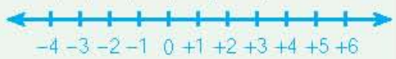
Classwork 1.3

Solution

(a) Arrange the following numbers in descending order.
–4, –5, +1, +5, 0, –1, +3

∴ ________________

(b) Arrange the following numbers in ascending order.
–1, 0, –2, +2, –4, +6, –3



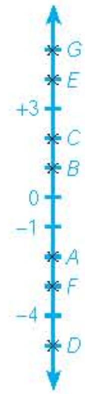
∴ ________________

Skills Upgrading Corner 1.2

Solution

1. **(a)** According to the number line on the right, find the values of A to G.

(b) Arrange the above numbers in ascending order.



Solution

2. The emergency room of a hospital is carrying out a patient sorting system. Patients will be treated in an order according to the seriousness of their conditions. It is given that the average waiting time for treatment is 94 minutes.

There are 5 patients waiting for treatment. The differences between their waiting time and the average waiting time are as follows (let +1 minute be 1 minute more than the average waiting time):


Edmond
0 minute


Stanley
−12 minutes


Janice
+50 minutes


Kenneth
+28 minutes


Denise
−20 minutes

Patient Sorting System at Emergency Room

Order of priority	Classification	Condition
1	Life-threatening	Major injuries, no heart beat
2	Emergency	Heart attack
3	Urgent	Seriously infected, broken bones
4	Semi-urgent	Fever, minor injuries
5	Non-urgent	Blocked nose, pimple

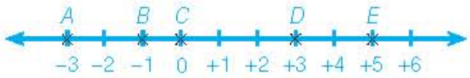
(a) Arrange their waiting time for treatment in ascending order. (Express your answer in positive or negative numbers.)

(b) Whose condition is the most serious? Whose condition is the least serious?

Exercise 1 B

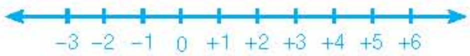
Level 1

1. Find the values of A to E on the following number line.



2. Mark points A to F on a number line.

   $A = -2$, $B = 1$, $C = 0$, $D = -1$, $E = 4$, $F = 3$



3. Find the larger number in each pair of numbers.

   (a) +3, +7　　**(b)** +4, −6
   (c) −3, +2　　**(d)** 0, −7
   (e) −5, −3　　**(f)** −8, −10

4. Find the smaller number in each pair of numbers.

   (a) +5, +4　　**(b)** +7, −6
   (c) −5, +5　　**(d)** −2, −3
   (e) −9, −11　　**(f)** 0, −6

5. Compare each group of numbers below, and fill in the blanks with '>' or '<'.

   (a) 18 ☐ 4　　**(b)** −5 ☐ −15
   (c) −6 ☐ 5　　**(d)** −2 ☐ 0
   (e) 6 ☐ −3　　**(f)** −10 ☐ 10

Level 2

6. **(a)** Mark points A to F on a number line.

   $A = +4$, $B = -6$, $C = +1$, $D = -1$, $E = +3$, $F = -7$

   (b) Arrange the above numbers in descending order.

7. Arrange the following numbers in ascending order.

(a) $-7, 2, -3, 5, 3, -2, 0$

(b) $4, 3.5, 0, -2, -3, -1.5, 2$

8. Simon and Eric are standing at the same place. If '+1 m' represents a move to the right by 1 m, on which side of Eric does Simon locate in each of the following situations?

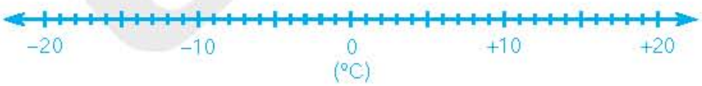
(a) Simon moves +5 m, Eric moves –4 m.

(b) Simon moves –5 m, Eric moves –3 m.

9. The daytime temperatures of some cities are as follows:

(a) If '+1°C' represents 1°C above zero, express the temperatures of the above cities in positive or negative numbers.

(b) Mark the temperatures of the cities on the following number line and arrange them in ascending order.



(c) If the daytime temperature of Hong Kong is 9°C above zero, how many cities are there with their temperatures higher than Hong Kong?

10. W, X, Y and Z are four points on a number line representing the values of w, x, y and z respectively. X lies on the right of Y, W lies on the left of Z, and Z lies on the right of X.

(a) Arrange w, x, y and z in ascending order. (There may be more than one answer.)

(b) When $x = 0$, find the possible values of w, y and z according to the result of **(a)**.

1.3 Addition and Subtraction of Directed Numbers

A Adding or subtracting positive numbers

Class Activity 1.2

Aim: To explore the directions of movement on a number line when adding and subtracting positive numbers

Joyce goes shopping at Timmy Department Store today.

1. **(a)** If '0' represents ground floor (G), '+1' represents the 1st floor, express the buttons of the lift in directed numbers.

Show Ans.

(b) The following table shows the shopping trip of Joyce to upper levels. Complete the table.

Show Ans.

	Original location	Shopping trip	Final location	Expression
(i)	5	Moves up 2 floors	7	$(+5) + 2 = +7$
(ii)	B1	Moves up 5 floors		
(iii)	B3	Moves up 3 floors		
(iv)	B2	Moves up 1 floor		

(c) The following table shows the shopping trip of Joyce to lower levels. Complete the table.

Show Ans.

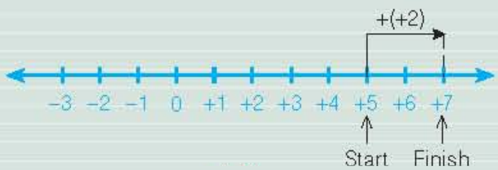
	Original location	Shopping trip	Final location	Expression
(i)	7	Moves down 3 floors	4	$(+7) - 3 = +4$
(ii)	3	Moves down 4 floors		
(iii)	G	Moves down 1 floor		
(iv)	B1	Moves down 2 floors		

2. **(a)** Since $2 = +2$, $(+5) + 2 = (+5) + (+2)$. Rewrite the expressions in **1(b)** in this way and mark the movements on the corresponding number lines. (The first one is an example.)

Show Ans.

(i) $(+5) + (+2) = +7$

Start Finish



(ii) ______ + ______ = ______

(iii) ______ + ______ = ______

(iv) ______ + ______ = ______

(b) Consider the number lines in **(a)**. When a positive number is added to a directed number, which direction does the directed number move to?

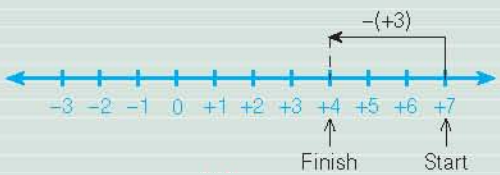
____

3. **(a)** Similarly, since $3 = +3$, $(+7) - 3 = (+7) - (+3)$. Rewrite the expressions in **1(c)** in this way and mark the movements on the corresponding number lines. (The first one is an example.)

Show Ans.

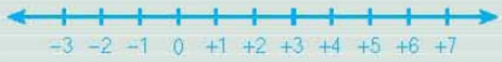
(i) $(+7) - (+3) = +4$

Start Finish



(ii) ______ − ______ = ______

(iii) ______ – ______ = ______



(iv) ______ – ______ = ______

(b) Consider the number lines in **(a)**. When a positive number is subtracted from a directed number, which direction does the directed number move to?

____

From the above Class Activity, we can obtain the rules of operation on a number line.

> Move to the right when adding a positive number.

e.g. $(-3)+(+7)=+4$

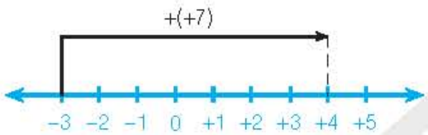


Figure 1.6

> Move to the left when subtracting a positive number.

e.g. $(+2)-(+8)=-6$

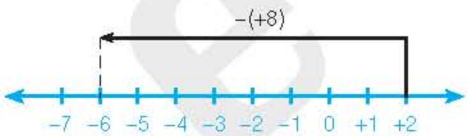


Figure 1.7

Additional Example 1.4

Example 1.4 Adding/subtracting a positive number

Find the values of the following expressions by using number lines.

(a) $(+3)+(+2)$

(b) $(+7)-(+9)$

(c) $(-5)+(+3)$

(d) $(-2)-(+7)$

Classwork 1.4

Solution

Find the values of the following expressions by using number lines.

(a) $(+5)+(+1)$

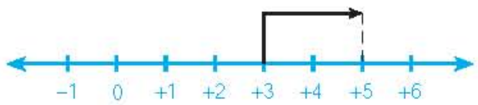
(b) $(+6)-(+10)$

(c) $(-4)+(+6)$

(d) $(-6)-(+6)$

Solution

(a)



Move 2 units to the right when adding +2.

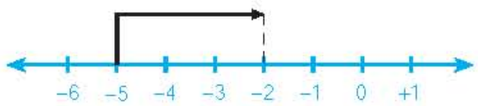
$\therefore\ (+3)+(+2)=\underline{\underline{+5}}$

(b)

Move 9 units to the left when subtracting +9.

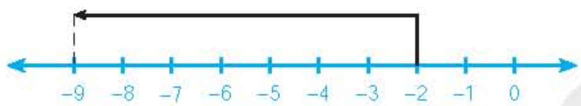
$\therefore\ (+7)-(+9)=\underline{\underline{-2}}$

(c)



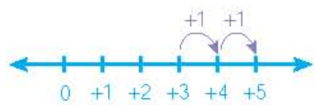
$\therefore\ (-5)+(+3)=\underline{\underline{-2}}$

(d)

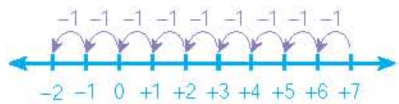


$\therefore\ (-2)-(+7)=\underline{\underline{-9}}$

Consider the expression of Example 1.4(a). Since the result of $(+3)+(+2)$ is equal to that of $(+3)+2$, $+(+2)=+2$. Likewise, consider the expression of Example 1.4(b). Since the result of $(+7)-(+9)$ is equal to that of $(+7)-9$, $-(+9)=-9$. To summarize, the rules for removing brackets are as follows:



$\therefore\ (+3)+2=+5$



$\therefore\ (+7)-9=-2$

$$A+(+B)=A+B$$
$$A-(+B)=A-B$$

Extension 1.2 Solution

Match the expressions on the left with those on the right.

$(+2)+(+1)$ •	• $2-1$
$(+2)-(+1)$ •	• $-2-1$
$(-2)+(+1)$ •	• $2+1$
$(-2)-(+1)$ •	• $-2+1$

B Adding or subtracting negative numbers

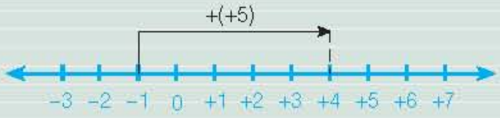
Class Activity 1.3

Aim: To explore the directions of movement on a number line when adding and subtracting negative numbers

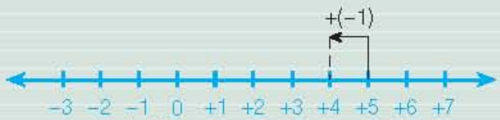
1. (a) Since different orders of an addition give the same result (e.g. $7+3=3+7$), $(-1)+(+5)=(+5)+(-1)$. Rewrite the following expressions in this way and mark the movements on the corresponding number lines. (The first one is an example.)

Show Ans.

(i) $(-1)+(+5)=+4$



$(+5)+(-1)=+4$



(ii) $(-3)+(+4)=$ ________

$(+4)+(-3)=$ ________

(iii) $(-3)+0=$ ________

$0+(-3)=$ ________

(b) Consider the number lines in **(a)**. When a negative number is added to a directed number, which direction does the directed number move to?

____

2. (a) Since $(-2)-(-2)=0$,

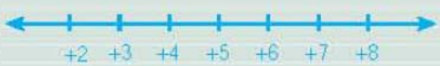
Show Ans.

we have $7+(-2)-(-2)=7$

________ $-(-2)=7$ .... (*)

(b) (i) If we mark the movement of the expression (*) on a number line, the starting point is ________ and the finishing point is ________ .

(ii) Mark the movement of the expression (*) on the following number line.



(c) When a negative number is subtracting from a directed number, the directed number moves to the left / right .

From the above Class Activity, we can obtain the rules for operation on a number line.

> Move to the left when adding a negative number.

e.g. $(+2)+(-5)=-3$

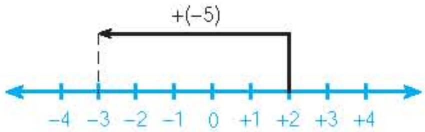


Figure 1.8

> Move to the right when subtracting a negative number.

e.g. $(-6)-(-4)=-2$

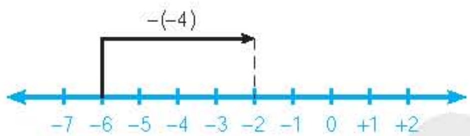


Figure 1.9

Additional Example 1.5

Example 1.5 Adding/subtracting a negative number

Find the values of the following expressions by using number lines.

(a) $(+3)+(-4)$

(b) $(+5)-(-3)$

(c) $(-1)+(-6)$

(d) $(-3)-(-1)$

Classwork 1.5

Solution

Find the values of the following expressions by using number lines.

(a) $(+3)+(-6)$

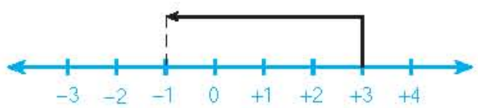
(b) $(+7)-(-8)$

(c) $(-2)+(-4)$

(d) $(-9)-(-4)$

Solution

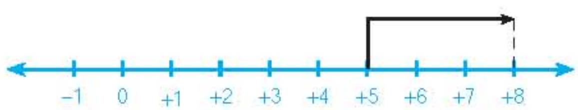
(a)



Move 4 units to the left when adding –4.

$\therefore\ (+3)+(-4)=\underline{\underline{-1}}$

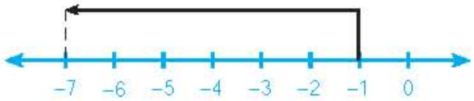
(b)



Move 3 units to the right when subtracting –3.

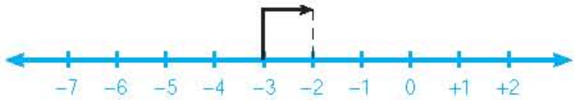
$\therefore\ (+5)-(-3)=\underline{\underline{+8}}$

(c)



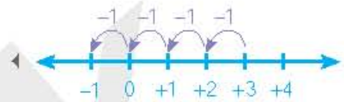
$\therefore\ (-1)+(-6)=\underline{\underline{-7}}$

(d)

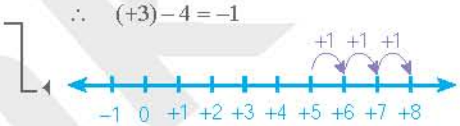


$\therefore\ (-3)-(-1)=\underline{\underline{-2}}$

Consider the expression of Example 1.5(a). Since the result of $(+3)+(-4)$ is equal to that of $(+3)-4$, $+(-4)=-4$. Likewise, consider the expression of Example 1.5(b). Since the result of $(+5)-(-3)$ is equal to that of $(+5)+3$, $-(-3)=+3$. To summarize, the rules for removing brackets are as follows:



$\therefore\ (+3)-4=-1$



$\therefore\ (+5)+3=+8$

$$A+(-B)=A-B$$
$$A-(-B)=A+B$$

Now, we can use these rules to simplify expressions.

Additional Example 1.6

Example 1.6 Addition or subtraction of two directed numbers

Simplify and evaluate the following.

(a) $(+4)+(+3)$

(b) $(-5)+(-2)$

(c) $(+4)-(+3)$

(d) $(-5)-(-2)$

Solution

(a) $(+4)+(+3)=4+3$
$=\underline{\underline{7}}$

(b) $(-5)+(-2)=-5-2$
$=\underline{\underline{-7}}$

(c) $(+4)-(+3)=4-3$
$=\underline{\underline{1}}$

(d) $(-5)-(-2)=-5+2$
$=\underline{\underline{-3}}$

Classwork 1.6

Solution

Simplify and evaluate the following.

(a) $(+5)+(+9)$

(b) $(-7)+(-2)$

(c) $(+6)-(+4)$

(d) $(+4)-(-9)$

Additional Example 1.7

Example 1.7 Addition or subtraction of three directed numbers

Simplify and evaluate the following.

(a) $(-3)+(-5)+(+8)$

(b) $(+10)-(-5)-(+9)$

(c) $(+5)+(-6)+(-9)$

(d) $(-12)-(+4)-(-5.5)$

Solution

(a) $\begin{aligned}(-3)+(-5)+(+8)&=-3-5+8\\&=-8+8\\&=\underline{\underline{0}}\end{aligned}$

(b) $\begin{aligned}(+10)-(-5)-(+9)&=10+5-9\\&=15-9\\&=\underline{\underline{6}}\end{aligned}$

(c) $\begin{aligned}(+5)+(-6)+(-9)&=5-6-9\\&=-1-9\\&=\underline{\underline{-10}}\end{aligned}$

(d) $\begin{aligned}(-12)-(+4)-(-5.5)&=-12-4+5.5\\&=-16+5.5\\&=\underline{\underline{-10.5}}\end{aligned}$

Classwork 1.7

Solution

Simplify and evaluate the following.

(a) $(-7)+(-6)+(+1)$

(b) $(+8)-(-3)-(+2)$

(c) $(+4)+(-9)+(-5)$

(d) $(-2)-(-7)-(+11.2)$

Additional Example 1.8

Example 1.8 Application of addition or subtraction of directed numbers

Daily temperature difference of a desert is very large. In *Gobi Desert*, which is located between China and Mongolia, the annual temperature is between 38°C and −47°C.

(a) What is the annual temperature difference of Gobi Desert?

(b) If the temperature drops from 37°C to −29°C, find the change in temperature.

(c) A weather station of the desert reported a cool weather in the morning. The temperature dropped by 46°C to −31°C in the evening. Find the temperature in the morning.

Classwork 1.8

Solution

A man is originally at the east of the bus stop. He walks 150 m to the west and stops at a location which is 35 m to the west of the bus stop. Find his original location.

Gobi Desert 戈壁沙漠

Solution

(a) Annual temperature difference

$= 38°C - (-47°C)$

$= 38°C + 47°C$

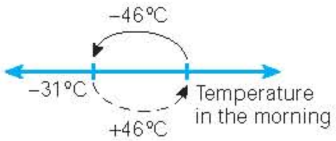
$= \underline{\underline{85°C}}$

(b) Change in temperature

= New temperature − Original temperature

$= -29°C - 37°C$

$= \underline{\underline{-66°C}}$

(c) [Analysis: 

]

Temperature in the morning

$= -31°C + 46°C$

$= \underline{\underline{15°C}}$

Skills Upgrading Corner 1.3

Solution **1.** Simplify and evaluate the following expressions.

(a) $0-(+2)-(-3)$

(b) $(-5)+(-8.3)+(+7)$

Solution **2.** Sam takes a lift from Basement 2 to the 9th floor. How many floors does he move up?

Solution **3.** The sum of $+7$ and a number is -5. Find the number.

Solution **4.** The sales records of a fruit shop for the past five days are as follows.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Profit / Loss	Profit \$500	Loss \$200	Balance	Profit \$250	Loss \$90
Remark					

What is the total profit/loss for these five days? Does the shopkeeper have an overall profit?

Exercise 1C

Level 1

Find the values of the following by using number lines. (**1** – **2**)

1. **(a)** $(+3)+(+7)$ **(b)** $(+4)-(+5)$ **(c)** $(-2)+(+3)$
(d) $(-6)+(+6)$ **(e)** $(-7)-(+2)$ **(f)** $(+2)-(+3)-(+4)$

2. **(a)** $(+4)+(-6)$ **(b)** $(+2)-(-5)$ **(c)** $(-4)+(-6)$
(d) $(-2)-(-3)$ **(e)** $(-4)-(-2)$ **(f)** $(+5)-(-4)-(-3)$

Evaluate the following. (**3** – **4**)

3. **(a)** $(+5)+(+5)$ **(b)** $(+3)-(-4)$ **(c)** $(-5)-(+3)$

4. **(a)** $(-7)+(+4)+(+5.2)$ **(b)** $(-5)+(-3.7)+(-1)$ **(c)** $(-2.4)-(-3)-(+4.7)$

5. The indoor temperature is 10°C and the outdoor temperature is –3°C. How much higher is the indoor temperature than the outdoor temperature?

6. The temperature was –2°C in the morning yesterday and it rose by 15°C in the afternoon. Find the temperature in the afternoon.

7. After diving 10 m below the sea level, the diver dives 5 m further. Express the diver's final location in a directed number.

8. Johnson walks 3 km to the east, and then 5 km to the west. If the east is the positive direction, express Johnson's final location in a directed number.

Level 2

9. The sum of –4 and a number is 8. Find the number.

10. The result of a number minus –2 is 5. Find the number.

11. Rocky went 9 floors up by a lift to the 6th floor. Which floor did he start going up?

12. The stored value of Kit's Octopus card is \$12. After buying a magazine, the remaining value of his card is –\$6. How much is the magazine?

13. The temperature of a city was –10°C yesterday morning.
(a) At noon, the temperature of the city went up by 6°C. Find the temperature of the city at noon.
(b) In the evening, the temperature of the city was –12°C. Comparing with the temperature at noon, find the change in temperature.

14. The following table shows the time differences between the HKSAR and some cities.

	Sydney	Tokyo	Paris	London
Time difference from the HKSAR local time (hour)	+2	+1	−7	−8

(a) If the HKSAR local time is 11 a.m., what is the time in the above cities?

(b) Your friend called you at 9 a.m. from London. What time was it in the HKSAR?

1.4 Mixed Addition and Subtraction of Directed Numbers

In the last section, we have discovered the following rules for removing brackets:

$$A + (+B) = A + B$$
$$A - (+B) = A - B$$
$$A + (-B) = A - B$$
$$A - (-B) = A + B$$

In this section, we will use these rules to simplify all expressions.

Additional Example 1.9

Example 1.9 Mixed addition and subtraction of directed numbers

Evaluate the following.

(a) $-5+(+3)-(-2)+(-4)$

(b) $(-3+4)-(-5+1)+(3-9)$

Solution

(a)
$$\begin{aligned}-5+(+3)-(-2)+(-4) &= -5+3+2-4\\ &= -2+2-4\\ &= 0-4\\ &= \underline{\underline{-4}}\end{aligned}$$

Classwork 1.9 Solution

Evaluate the following.

(a) $-7+(-3)-(+5)-(-4)$

(b) $(-7-3)+(9-2)-(-2+6)$

(b) $(-3+4)-(-5+1)+(3-9) = 1-(-4)+(-6)$
$= 1+4-6$
$= 5-6$
$= \underline{\underline{-1}}$

Additional Example 1.10

Example 1.10 Mixed addition and subtraction of directed numbers with brackets

Evaluate the following.

(a) $5-[(-3)-(-2)]$

(b) $10-[(-4+2)+(2-5)]$

Solution

(a) $5-[(-3)-(-2)] = 5-(-3+2)$
$= 5-(-1)$
$= 5+1$
$= \underline{\underline{6}}$

(b) $10-[(-4+2)+(2-5)] = 10-[(-2)+(-3)]$
$= 10-(-2-3)$
$= 10-(-5)$
$= 10+5$
$= \underline{\underline{15}}$

Classwork 1.10

Solution

Evaluate the following.

(a) $-4-[6-(-8)]$

(b) $-7-[(5-12)-(-3)]$

Additional Example 1.11

Example 1.11 Application of mixed addition and subtraction of directed numbers

A submarine moves 100 m down from 400 m below the sea level, then moves 50 m up. Express the final location of the submarine in a directed number.

Solution

The final location $= (-400-100+50)$ m
$= (-500+50)$ m
$= \underline{\underline{-450 \text{ m}}}$

Classwork 1.11

Solution

Charles takes a lift at the ground floor. He goes 6 floors up, and goes 1 floor down. He goes 3 floors up again and goes 9 floors down finally. What is his final location?

Skills Upgrading Corner 1.4

Solution

1. Evaluate the following.
   (a) $-6-(2-5)+(4-8)$
   (b) $-9-(-11)+5-[-4-(-20+13)]$

Solution

2. The average weight of S1A students is 45 kg. Nancy, Andy, Miki and Doris are students in this class, and their weights are 47 kg, 50 kg, 65 kg and 38 kg respectively.
   (a) If Nancy's weight is +2 kg in comparison with the average weight, what is the weight of Andy, Miki and Doris each in comparison with the average weight?
   (b) Amy is also an S1A student. If Amy's weight is –4 kg in comparison with the average weight, by how much is Amy heavier than Doris?

Exercise 1 D

Level 1

Evaluate the following. (**1** – **3**)

1. (a) $1+5-7$ (b) $-7+12-9$
   (c) $-12-5+8$ (d) $5-8+6-3$

2. (a) $(+3)+(-5)-(+8)$ (b) $(-6)-(-9)+(+7)$
   (c) $(+2)+(+8)-(+9.2)+(-11)$ (d) $-3-9.4+(-4)-7-(+6.2)$

3. (a) $0-(-3+5)$ (b) $-1-[(-5)-(-7)]$
   (c) $2+[\frac{2}{3}-(-7+4)]$ (d) $-12+[10-(-8\frac{3}{4})-25]-(2-6\frac{1}{4})$

4. A submarine moves 50 m up from 100 m below the sea level, and then moves 80 m down. Express the final location of the submarine in a directed number.

5. One day, Kevin forgot to bring his wallet. In the morning, he borrowed \$12 from Vivian for breakfast. In the afternoon, he borrowed \$22 from her again for lunch. If Kevin returned \$20 to Vivian the next day, how much was he still owing Vivian?

6. The score of Woody in a golf competition is as follows:

Round	1	2	3	4
Score	3 strokes below the standard	4 strokes above the standard	2 strokes below the standard	1 stroke below the standard

Express the total score of Woody in a directed number.

Level 2

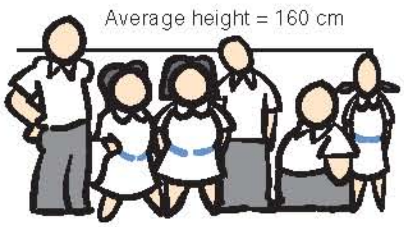
7. Fiona, Ella and Gigi joined a game called 'Yummy Kitchen'. In the game, they cooked a dish each and their scores were given by five judges. The one with the highest score could win a set of kitchenware. Their scores are as follows:

Who won the set of kitchenware? Explain briefly.

8. The average height of Kate, Emma, Tom, Mike, Jane and Alan is 160 cm.

(a) Complete the following table.

Name	Kate	Emma	Tom	Mike	Jane	Alan
Height (cm)	158			156		164
Difference from the average height (cm)	−2	−1	+3		0	



(b) Who is the tallest? Who is the shortest?

9. Mark's monthly salary is \$20 000. He has a *surplus* if he spends less than his salary; he has a *deficit* if he spends more than his salary. His surplus and deficit for the first half of the year are shown in positive and negative numbers respectively in the table below.

Month	January	February	March	April	May	June
Surplus/Deficit (\$)	+2 000	+3 000	−3 500	+1 500	−6 000	+2 500

(a) Which month did Mark spend the least and how much did he spend in that month?

(b) Which month did Mark spend the most and how much did he spend in that month?

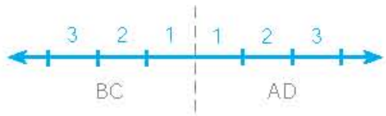
surplus 盈餘　　deficit 赤字

10. The following table shows the cash and bills of Sam at different stages and their corresponding changes in status. Let cash owned be a positive value, and a bill payable be a negative value. Complete the following table.

Cash and bill	Amount (\$)	Change in status	Expression (\$)	Balance (\$)
Cash \$100, Cash \$300, Bill \$150, Bill \$30	+220	Pay Bill \$30	+220 − (−30)	+250
Cash \$80, Bill \$120, Cash \$160, Bill \$70		Pay Cash \$80		
Bill \$300, Bill \$130, Cash \$250, Cash \$120		Pay Bill \$130		
Bill \$360, Cash \$540, Bill \$270, Cash \$110	+20	Receive Cash \$100		+120
Cash \$530, Cash \$320, Bill \$470, Bill \$210		Receive Cash \$100		
Cash \$250, Cash \$120, Bill \$150, Bill \$300		Receive Bill \$100		

11. The following table shows some *dynasties* in China and their corresponding periods under the western calendar.

There is nothing called 'AD 0' under the western calendar.



Dynasty	Period
Qin Dynasty	221 BC - 206 BC
Western Han Dynasty	206 BC - AD 8
Eastern Han Dynasty	AD 25 - AD 220
Six Dynasties	AD 220 - AD 581
Sui Dynasty	AD 581 - AD 618
Tang Dynasty	AD 618 - AD 907
Five Dynasties	AD 907 - AD 960
Northern Song Dynasty	AD 960 - AD 1127
Southern Song Dynasty	AD 1127 - AD 1279
Yuan Dynasty	AD 1279 - AD 1368
Ming Dynasty	AD 1368 - AD 1644
Qing Dynasty	AD 1644 - AD 1911

dynasty 朝代

(a) Design a model to express the period of each dynasty in directed numbers.

(b) How long is the duration of the Western Han Dynasty?
[e.g. Duration of the Qing Dynasty = (1 911 − 1 644 + 1) years = 268 years]

(c) How many years are there from the beginning of the Qin Dynasty to the fall of the Qing Dynasty?

(d) Which dynasty started after 1 000 years from the fall of the Qin Dynasty?

1.5 Multiplication and Division of Directed Numbers

A Multiplication of directed numbers

Class Activity 1.4

Aim: To explore the meaning of multiplication of two directed numbers

Tool required: A pair of scissors

Cut two types of cards, $\boxed{-10}$ and $\boxed{+10}$, of 6 copies each from the Appendix and put 3 each on a table. (See Figure I)

Figure I

1. Given that these two types of cards represent different scores, the total score obtained from the 6 cards on the table is ________.

Show Ans.

2. Put down 2 more $\boxed{+10}$ cards, and express this as $(+2)\times(+10)$. (See Figure II)

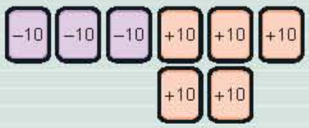


Figure II

The total score obtained from all cards on the table now is ________.

Thus, $(+2)\times(+10)$ = ________.

Show Ans.

3. Put down 3 more $\boxed{-10}$ cards, and express this as $(+3)\times(-10)$. (See Figure III)

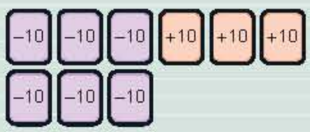


Figure III

The total score obtained from all cards on the table now is ________.

Thus, $(+3)\times(-10)$ = ________.

Show Ans.

4. Take away 2 [+10] cards, and express this as $(-2)\times(+10)$. (See Figure IV)

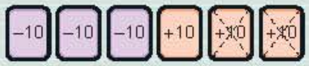


Figure IV

The total score obtained from all cards on the table now is ________.

Thus, $(-2)\times(+10) =$ ________.

Show Ans.

5. Take away 3 [−10] cards, and express this as $(-3)\times(-10)$. (See Figure V)

Figure V

The total score obtained from all cards on the table now is ________.

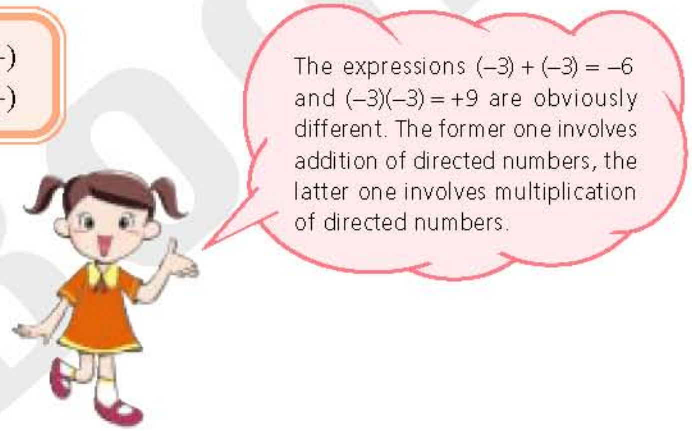
Thus, $(-3)\times(-10) =$ ________.

Show Ans.

From the above Class Activity, the rules for multiplication of directed numbers are obtained as follows.

I.T. Tool

$(+)(+) = (+)$	$(+)(-) = (-)$
$(-)(-) = (+)$	$(-)(+) = (-)$



Example 1.12 Multiplication of directed numbers

Evaluate the following.

(a) $(+2)(+4)$ **(b)** $(+2)(-7)$

(c) $(-3)(+8)$ **(d)** $(-3)(-5)$

Solution

(a) $(+2)(+4) = +(2\times4)$
$= \underline{\underline{8}}$

(b) $(+2)(-7) = -(2\times7)$
$= \underline{\underline{-14}}$

(c) $(-3)(+8) = -(3\times8)$
$= \underline{\underline{-24}}$

(d) $(-3)(-5) = +(3\times5)$
$= \underline{\underline{15}}$

Additional Example 1.12

Classwork 1.12

Solution

Evaluate the following.

(a) $(+6)(+3)$

(b) $(+5)(-5)$

(c) $(-3)(+7)$

(d) $(-2)(-9)$

B Division of directed numbers

Class Activity 1.5

Aim: To explore the rules for division of directed numbers

1. Evaluate the following. Show Ans.

(a) $(+5)(+2) =$ ______ (b) $(+5)(-2) =$ ______

(c) $(-5)(+2) =$ ______ (d) $(-5)(-2) =$ ______

2. It is given that $3 \times 8 = 24$ can be rewritten as $24 \div 8 = 3$. Rewrite the expression of question **1** in this way. Show Ans.

(a) ______ $\div$ ______ $= +5$ (b) ______ $\div (-2) =$ ______

(c) ______ $\div$ ______ $= -5$ (d) ______ $\div$ ______ $=$ ______

3. According to the results above, complete the following rules for division of directed numbers. Show Ans.

$(+) \div (+) =$ ______ $(+) \div (-) =$ ______

$(-) \div (-) =$ ______ $(-) \div (+) =$ ______

From the above Class Activity, the rules for division of directed numbers are obtained as follows.

$$\frac{(+)}{(+)} = (+) \qquad \frac{(+)}{(-)} = (-)$$
$$\frac{(-)}{(-)} = (+) \qquad \frac{(-)}{(+)} = (-)$$

Additional Example 1.13

Example 1.13 Division of directed numbers

Evaluate the following.

(a) $(+12) \div (+3)$

(b) $(+54) \div (-6)$

(c) $\dfrac{-72}{+4}$

(d) $\dfrac{-63}{-9}$

Classwork 1.13

Solution

Evaluate the following.

(a) $(+16) \div (+4)$ (b) $(+5) \div (-5)$

(c) $(-9) \div (+3)$ (d) $(-6) \div (-6)$

(e) $\dfrac{+76}{-4}$ (f) $\dfrac{-150}{-25}$

Solution

(a) $(+12)\div(+3)=+\frac{12}{3}$
$=\underline{\underline{4}}$

(b) $(+54)\div(-6)=-\frac{54}{6}$
$=\underline{\underline{-9}}$

(c) $\frac{-72}{+4}=-\frac{72}{4}$
$=\underline{\underline{-18}}$

(d) $\frac{-63}{-9}=+\frac{63}{9}$
$=\underline{\underline{7}}$

Additional Example 1.14

Example 1.14 Mixed multiplication and division of directed numbers

Evaluate the following.

(a) $\frac{(-4)(-3)(-5)}{(+6)(-15)}$

(b) $(-12)\div(-\frac{1}{2})\times(-\frac{1}{3})\div(+4)$

Classwork 1.14 Solution

Evaluate the following.

(a) $\frac{(-2)(-6)}{(-5)(+4)(-9)}$

(b) $(-8)\div(-\frac{1}{3})\times(+\frac{1}{4})\div(-6)$

(c) $(-8)\times 2\div 64\div(-1)$

Solution

(a) $\frac{(-4)(-3)(-5)}{(+6)(-15)}=\frac{[+(4\times 3)](-5)}{-(6\times 15)}$

Handle the signs of the numerators and denominators before finding the value.

$=\frac{-(4\times 3\times 5)}{-(6\times 15)}$
$=+\frac{4\times 3\times 5}{6\times 15}$
$=\underline{\underline{\frac{2}{3}}}$

(b) $(-12)\div(-\frac{1}{2})\times(-\frac{1}{3})\div(+4)=\frac{(-12)(-\frac{1}{3})}{(-\frac{1}{2})(+4)}$
$=\frac{+(12\times\frac{1}{3})}{-(\frac{1}{2}\times 4)}$
$=-\frac{4}{2}$
$=\underline{\underline{-2}}$

Additional Example 1.15

Example 1.15 Directed numbers under the four basic operations of arithmetic

Evaluate the following.

(a) $(-3)\times(+4)-(-18)\div(+2)$

(b) $\dfrac{2+[(-11)-(-5)]}{-6-(-4)-8}$

Solution

(a) $(-3)\times(+4)-(-18)\div(+2)=(-12)-(-9)$
$=-12+9$
$=\underline{\underline{-3}}$

Do multiplications and divisions before additions and subtractions.

(b) $\dfrac{2+[(-11)-(-5)]}{-6-(-4)-8}=\dfrac{2+(-11+5)}{-6+4-8}$
$=\dfrac{2+(-6)}{-2-8}$
$=\dfrac{2-6}{-10}$
$=\dfrac{-4}{-10}$
$=\underline{\underline{\dfrac{2}{5}}}$

Classwork 1.15

Solution

Evaluate the following.

(a) $(-8)\div(-2)+(+4)\times(-6)$

(b) $\dfrac{(-7)\times[8-(-7)]}{(+4-18)\times(-5)}$

Additional Example 1.16

Example 1.16 Application of directed numbers under the four basic operations of arithmetic

There are 10 multiple choice questions in a Mathematics quiz. 5 marks are given for each correct answer, 3 marks are deducted for each wrong answer, 1 mark is deducted for each question left blank, and thus the quiz can end up in negative marks. Wing has answered 6 questions correctly, 2 questions wrongly, and left 2 questions blank. Sally has answered 7 questions correctly and 3 questions wrongly. What is the sum of their marks?

Solution

Wing's marks $=[6\times(+5)+2\times(-3)+2\times(-1)]$ marks
$=(30-6-2)$ marks
$=22$ marks

Sally's marks $=[7\times(+5)+3\times(-3)]$ marks
$=(35-9)$ marks
$=26$ marks

$\therefore$ Sum of their marks $=(22+26)$ marks
$=\underline{\underline{48\text{ marks}}}$

Classwork 1.16

Solution

There are 20 multiple choice questions in an English test, and its marking scheme is as follows:

Correct	Wrong	Blank
+5 marks each	−3 marks each	0 mark each

* Final score below 0 will be treated as 0 mark.

(a) Amy has answered all the questions and got 7 of them wrong. Find her marks.

(b) Jay has answered all the questions and got 4 of them correct. Find his marks.

(c) Herman has answered 14 questions and got 7 of them wrong. Find his marks.

(d) Susan has answered 8 questions and got 4 of them correct. Find her marks.

Skills Upgrading Corner 1.5

Solution **1.** Evaluate the following.

(a) $(+2)+(-4)(-8)-(+9)(-3)$

(b) $[(-140)\div(-7)-(-2)]\div[-9-(+2)]$

Solution **2.** There are 10 questions in a *psychological test*. Each question has 3 choices. The marking scheme and the analysis are as follows:

Choice	Mark
A	+3
B	−1
C	−3

Total marks	Character
From 18 to 30	Active and outgoing
From −10 to 17	Gentle and lively
From −30 to −11	Quiet and shy

The choices of Edward and Kelly are as follows:

Edward: 7 A's, 2 B's and 1 C's

Kelly: 3 A's, 3 B's and 4 C's

(a) Find the marks of each of them.

(b) According to the above test, what is the respective character of Edward and Kelly?

Exercise 1E

Level 1

1. Complete the following multiplication table.

×)	+5	+3	+1	0	−2	−4	−6
+5	+25	+15	+5	0			
+3				0			
+1				0			
0				0			
−2				0			
−4				0			
−6				0			

psychological test 心理測驗

2. Complete the following division table.

b \ a ($\frac{a}{b}$)	$+12$	$+6$	0	-6	-12
$+3$	$+4$	$+2$	0		
$+2$	$+6$	$+3$	0		
$+1$			0		
-1			0		
-2			0	$+3$	$+6$
-3			0	$+2$	$+4$

Evaluate the following. (**3** – **6**)

3. **(a)** $(+2)(+6)$ **(b)** $(+10)(+15)$ **(c)** $(+7)(-9)$

(d) $(-5)(+20)$ **(e)** $(-8)(-6)$ **(f)** $(-3)(-12)$

4. **(a)** $(+35)\div(+5)$ **(b)** $(+48)\div(-2)$ **(c)** $(-32)\div(+8)$

(d) $(-24)\div(-3)$ **(e)** $\dfrac{+126}{-9}$ **(f)** $\dfrac{-242}{-11}$

5. **(a)** $(+3)(-2)(+5)$ **(b)** $(-2)(-4)(-6)$ **(c)** $\dfrac{(-15)(+2)}{-10}$

(d) $\dfrac{-60}{(+15)(-4)}$ **(e)** $\dfrac{(-4)(-4)(-15)}{(-8)(+5)}$ **(f)** $\dfrac{(+12)(-18)(-40)}{(-36)(+5)(-48)}$

6. **(a)** $(-24)\div(-3)\div(-2)$ **(b)** $(-6)\times(-\frac{1}{2})\div(-\frac{1}{3})$ **(c)** $(-\frac{1}{2})\div(+2)\times(-4)$

(d) $(+2)\times(-10)\div(+4)\times(-\frac{1}{5})$ **(e)** $(+7)\div(-\frac{7}{2})\div(-2)\div(+\frac{1}{4})$ **(f)** $-(-5)\div(-\frac{1}{3})\div(+\frac{3}{4})\times(-2)$

Level 2

Evaluate the following. (**7** – **9**)

7. **(a)** $(-2)(-10)+(-4)(-5)$ **(b)** $(+8)\div(-4)-(+8)\div(-2)$

(c) $(-3)\times(+2)-(+6)\div(-3)$ **(d)** $(+15)\div(+5)+(-6)\times(-3)$

8. **(a)** $(+7)+(-3)(+2)+(+3)(-3)$ **(b)** $(-8)-(+3)(-2)-(-4)(+5)$

(c) $(-3)-(-2)(+3)-(-3)(-4)(-5)$ **(d)** $(-7)(-3)+(-4)(-5)-(+3)(-2)(-5)$

9. **(a)** $\dfrac{(+8)-(-4)(+3)}{(+2)(-2)}$ **(b)** $\dfrac{[(+3)-(-4)](-5)}{(+7)(-2)}$

(c) $[(-15)+(-3)(+5)]\div[(-4)(-3)]$ **(d)** $[(-2)(+3)-(-2+3)]\div[(+2)(-3)]$

10. Mr. Lee sells 250 pieces of Item A at a profit of \$5 each, and 300 pieces of Item B at a profit of \$12 each, but 420 pieces of Item C at a loss of \$20 each. What is the overall profit or loss made by Mr. Lee?

11. The marking scheme of a competition is as follows:

> *First attempt*
> - 20 marks are given for each correct answer.
> - 10 marks are deducted for each wrong answer.
>
> *Second attempt*
> - 10 marks are given for each correct answer.
> - 5 marks are deducted for each wrong answer.

The results of the competition of S1A and S1B are as follows:

	First attempt		Second attempt	
	Number of correct answers	Number of wrong answers	Number of correct answers	Number of wrong answers
S1A	15	7	3	3
S1B	14	6	5	2

(a) How many marks does each class get?

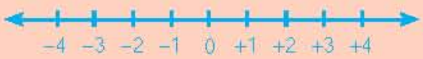
(b) Which class gets a higher mark?

Chapter Summary

Chapter Summary

A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

Column A			Column B
1. Positive numbers	•	•	**(a)** Numbers smaller than zero.
2. Negative numbers	•	•	**(b)** 
3. Directed numbers	•	•	**(c)** Numbers greater than zero.
4. Number line	•	•	**(d)** Numbers with positive or negative signs.

B. Fact to Remember

1. Rules for removing brackets

A + (+B) = A + B

A − (+B) = A − B

A + (−B) = A − B

A − (−B) = A + B

2. Rules for multiplication and division of directed numbers

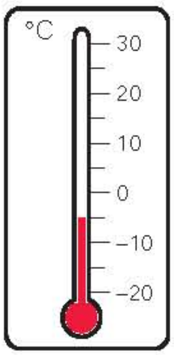
Rules for multiplication	$(+)(+) = (+)$ $(-)(-) = (+)$	$(+)(-) = (-)$ $(-)(+) = (-)$
Rules for division	$\frac{(+)}{(+)} = (+)$ $\frac{(-)}{(-)} = (+)$	$\frac{(+)}{(-)} = (-)$ $\frac{(-)}{(+)} = (-)$

Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

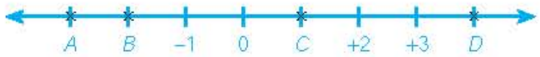
1. The thermometer shows the temperature of a city in a morning. 1.1



(a) The temperature in that morning was ________________ .

(b) Given that the temperature decreased by 4°C at night, what was the change in temperature?

2. **(a)** According to the number line below, find the values of A to D. 1.2



$A =$ ________, $B =$ ________,

$C =$ ________, $D =$ ________

(b) Mark the following numbers on a number line and then arrange them in descending order.

$A = -3$, $B = 2$, $C = 0$, $D = -1.5$, $E = 3$, $F = -0.5$

3. **(a)** Find the value of $(-2)+(+6)$ by using a number line. 1.3A

$(-2)+(+6) =$ __________

(b) Find the value of $(-4)-(+5)$ by using a number line.

4. **(a)** Find the value of $(-3)+(-4)$ by using a number line. 1.3B

$(-3)+(-4) =$ __________

(b) Find the value of $(-6)-(-7)$ by using a number line.

5. **(a)** $(+1)+(+5)-(-3) =$ __________ 1.4

(b) Evaluate $(-4)-[(+4)+(-6)]$.

6. **(a)** $(+3)(-2) =$ __________ 1.5A

(b) Evaluate $(-4)(-3)$.

7. **(a)** $(+8)\div(-2) =$ __________ 1.5B

(b) Evaluate $\dfrac{-3}{-6}$.

Revision Exercise 1

Level 1

1. The profit/loss records of Fantasy Florist for the first six months of this year are as follows. Complete the following table.

Month	Profit / Loss	Record
January	Loss \$2 000	−\$2 000
February		+\$3 000
March	Loss \$2 100	
April		−\$500
May	Profit \$3 200	
June	Balance	

2. Compare each group of numbers below and fill in the blanks with '>' or '<'.

(a) $-5 \ \square \ -2$ (b) $+8 \ \square \ -10$

(c) $0 \ \square \ -6$ (d) $0.5 \ \square \ 0$

(e) $-\frac{1}{3} \ \square \ \frac{1}{3}$ (f) $-\frac{4}{5} \ \square \ -\frac{2}{5}$

3. Fill in the blanks with suitable directed numbers.

(a) $(\qquad)+17=15$ (b) $-5+(\qquad)=-4$

(c) $(\qquad)+3=-8$ (d) $9-(\qquad)=-17$

(e) $(-5)(\qquad)=75$ (f) $(\qquad)(-12)=-108$

(g) $\dfrac{(\qquad)}{+8}=-13$ (h) $\dfrac{-243}{(\qquad)}=27$

Evaluate the following. (**4** – **6**)

4. (a) $(+6)+(+5)-(+3)$ (b) $(-7)-(-5)+(+4)$

(c) $(-9.7)-[(+2.4)-(-8)]$ (d) $[(-6\frac{1}{7})-(+2)]-[(-8)+(-12\frac{3}{7})]$

5. (a) $(-8)(-8)(-2)$ (b) $(-6)(-9)(+3)$

(c) $\dfrac{(+12)(-7)}{-2}$ (d) $\dfrac{(+7)(+2)(-8)}{(+14)(-4)}$

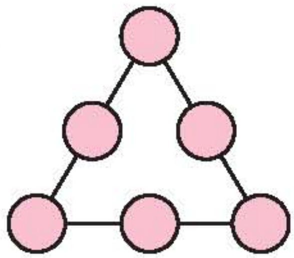
6. **(a)** $(-54)\div(+2)\div(-3)$ **(b)** $(-56)\div(-4)\times(+7)$
(c) $(-256)\div(+16)\div(-2)\div(-8)$ **(d)** $(-32)(+3)\div(-24)(-2)$

Level 2

7. Arrange each group of numbers below in ascending order.
(a) $-1,\ 8,\ -4,\ 4,\ 2$ **(b)** $0.3,\ 0.7,\ -0.3,\ -0.7,\ 0$
(c) $-3,\ -\frac{1}{2},\ \frac{5}{2},\ 0,\ -1,\ -\frac{3}{2}$ **(d)** $0.5,\ -5,\ -1.5,\ -1,\ 1.5,\ -3.5$

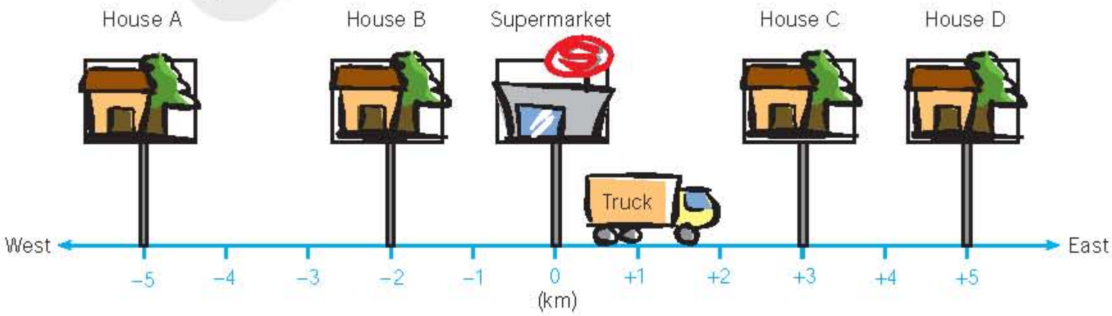
8. Evaluate the following.
(a) $(-8)\times(+6)\div[(+8)-(+5)]$ **(b)** $(+11)-(-2)\times[(+6)\div(-3)+(-2)]$
(c) $\dfrac{(+12)+[(-5)-(-3)]}{(-3+2)(-6)}$ **(d)** $[(-5)(-2)-(+3)(+4)]\div[(-6)+(-4)]\times(+2)$

9. In the figure, fill in the circles with -2, -1, 0, 1, 2 and 3, so that the sum of the numbers along each straight line is zero.



10. In a competition, 20 marks were given for each correct answer, 10 marks were deducted for each wrong answer, and 5 marks were deducted for each question given up. Jimmy answered 3 questions correctly, 5 questions wrongly, and gave up the rest. If Jimmy got 0 mark at the end, how many questions were provided for him in the competition?

11. The figure shows the locations of the supermarket, houses A, B, C and D. A truck leaves the supermarket and delivers the goods to houses C, D, A and then B.



(a) How far is the final location of the truck from the supermarket?
(b) Which two houses are the farthest from the supermarket?
(c) Which two houses are the closest to each other?
(d) After delivering the goods to house B, the truck goes back to the supermarket. Find the total distance travelled by the truck.

12. At the beginning of the year, Healthy Company had a cash balance of \$180 000. The company made a loss of \$40 000 monthly for the first five months, and then started making a profit of \$15 000 monthly for the rest of the year.

In the same year, Lively Company had a cash balance of \$200 000 at the beginning. The company made a profit of \$5 000 monthly for the first three months, and then started making a loss of \$30 000 monthly for the rest of the year.

(a) At the end of April, how much cash was left with Healthy Company and Lively Company each?

(b) At the end of August, which company had more cash left?

(c) Cheerful Company had a cash balance of \$220 000 at the beginning of the year. At the end of August, the cash balance of the company is the same as that of Lively Company. If Cheerful Company made the same amount of profit/loss monthly for the whole year, what was the amount of monthly profit/loss made by the company?

13. **(a)** **(i)** If '−1 m' represents 1 m to the west of a tree, express 10 m to the east of a tree in a positive or negative number.

(ii) If '+1 hour' represents 1 hour after noon, express 5 hours before noon in a positive or negative number.

(iii) If '+1 m/h' represents walking to the east at a speed of 1 m/h, express walking to the west at a speed of 6 m/h in a positive or negative number.

◂ Speed = $\dfrac{\text{Distance}}{\text{Time}}$

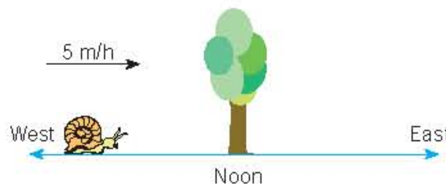
(b) A snail walked to the east all day at a speed of 5 m/h and passed the tree at noon. Express its locations at the following times in positive or negative numbers.

(i) 2 p.m.

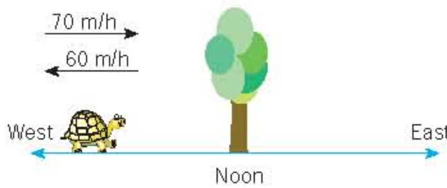
(ii) 5 p.m.

(iii) 10 a.m.

(iv) 7 a.m.



(c) A tortoise walked to the east at a speed of 70 m/h from 9 a.m. and arrived at the tree at noon. Then it walked to the west at a speed of 60 m/h until 3 p.m. Find the distance between its starting and final locations.



MC Question

14. Which of the following is true?

A. $0 > -1$

B. $-5 > -2$

C. $3 < -3$

D. $1 < 0$

15. In the figure, R is a point on the number line. Which of the following must **NOT** be the value of R?

A. −4

B. −3

C. −2.5

D. −1.5

16. According to the following figure, find the value of A.

A. -1
B. -3
C. -6
D. -9

17. $(-1)(+1)+(+1)(-1)-(-1)=$

A. $+1$.
B. 0.
C. -1.
D. -3.

18. Which of the following expressions has a positive value?

A. $(-\frac{1}{2})(+\frac{1}{4})(-5\frac{1}{2})(-6)$
B. $\frac{(-4)(-6)}{-5}$
C. $\frac{-4}{(-5)(-6)}$
D. $\frac{(+4)(-8)(+20)}{-6}$

19. Which of the following expressions has a value that is **NOT** equal to the value of $-(\frac{1}{-2})+(\frac{-2}{3})$?

A. $\frac{1}{2}-\frac{2}{3}$
B. $-(\frac{-1}{2})+(\frac{2}{-3})$
C. $-(\frac{1}{-2})-\frac{2}{3}$
D. $-(\frac{1}{-2})+\frac{2}{3}$

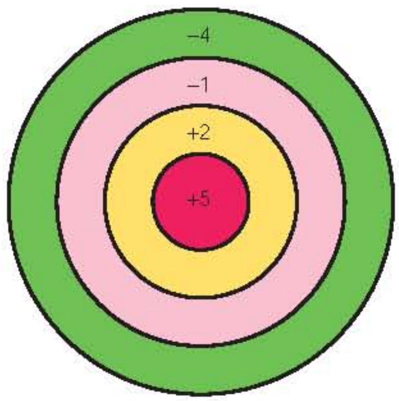
Problem-solving and Exploring

Hint for the Title Page Question

(a) If the three darts thrown by Patrick hit the regions +5, −1 and −4, what is his total score?

(b) In the following table, list all cases in getting a total score of 0.

+5	+2	−1	−4	Total score
1	0	1	1	0



Additional Question

1. 4 people played cards for 5 rounds. In each round, there is 1 winner, and the marks got by the winner equals the total marks lost by the losers. It is known that the marks lost and got are represented by a negative number and a positive number respectively. The following table shows the accumulated marks at the end of each round.

Participant / Round	Mike	John	Danny	Stephen
1	−5	−4	[redacted]	+33
2	−13	−11	−3	+27
3	−21	−17	+47	−9
4	−43	+49	+25	−31
5	−50	+89	+1	−40

(a) Who was the winner in the 1st round? How many marks did Danny lose?

(b) Who was the winner in the 2nd round? How many marks did he get?

(c) How many marks did Stephen lose in the 3rd round?

(d) How many rounds did each of them win?

2. A passenger paid a taxi fare of \$200 by a \$1 000 note. Since the taxi driver did not have any small change, he changed the \$1 000 note with a shop nearby and gave \$800 change to the passenger. The shopkeeper found that the \$1 000 note was fake right after the passenger had left, and he immediately got a real one back from the taxi driver. How much did each of them gain or lose as a whole? Explain briefly.